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Effects of multiple representations-based instruction on junior high schoolstudents' achievement in linear equations in one variable

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Abstract

Linear equations permeate many essential components of mathematical knowledge, yet the West African Examination Council report shows that the majority of students are unable to solve them adequately. In response to this, dual-representation-based learning has been introduced to minimize student difficulties, thus fostering good academic achievement. This study aims to examine the influence of dual-representation-based learning on middle school students' academic achievement in single-variable linear equations. The research employs a quasi-experimental design with a population of 159 students and a sample of 53 students obtained through convenience sampling. Data collection techniques include questionnaires and the Linear Equations Achievement Test (LEAT) to measure students' academic achievement. SPSS software is used in the research data analysis technique. The results show a statistically significant difference in student scores on the linear equation achievement test when using dual-representation-based instruction compared to single-representation instruction. It also indicates that a majority of middle school mathematics instructors (45.3%) consistently use a single representational learning approach in their linear equation pedagogy. However, a small fraction of teachers (14.0%) regularly incorporate multiple representations into their teaching. This suggests that dual-representation-based learning influences middle school students' academic achievement in single-variable linear equations. Research on dual-representation-based learning in single-variable linear equations can provide valuable insights for further understanding more effective and relevant learning methods in the context of middle school mathematics education.

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INTRODUCTION

The indispensable role of mathematics in the advancement of any society cannot be overemphasized. Sherrod et al., (2009) asserted that nations paying insufficient attention to mathematics studies are predisposed to lag in technological progression. No societal growth can occur without the influence of mathematics. Sacco (2013) contends that mathematics is an instrumental tool for problem-solving and modeling, further underscoring its ubiquity. According to Maclver (1931), a society comprises individuals sharing a common culture within a defined territory.

Every facet of human society harnesses the utility of mathematics. Its role is undeniably pivotal in vocational sectors, including carpentry, masonry, tailoring, and culinary arts, where the application of mathematical principles is inevitable. Furthermore, sectors such as banking, music, shopkeeping, and business necessitate a firm grasp of mathematical knowledge. In terms of infrastructure development, mathematics serves as the backbone, aiding in the construction of roads, buildings, and enabling mechanical and electrical engineering. Mathematics is a cornerstone of medical advancements, enabling cardiac modeling, DNA sequencing, gene technology, and the production of medical equipment (Fletcher, 2018).

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Algani (2022) elucidates that a society is an amalgamation of individuals and for societal progression, individual growth is paramount. Education is the tool for equipping and developing these individuals. Hence, it's no surprise that mathematics, due to its practicality, is a compulsory subject globally, including in Ghana. It fosters critical thinking, problem-solving, and innovation. Therefore, a pass in mathematics is a prerequisite for admission into second circle schools and tertiary institutions in Ghana.

Linear equations serve as a crucial component of mathematical knowledge. These equations permeate numerous topics within the field (Tanvir & Nieves, 2015) and persist from primary schooling through to university (Kumari & Poonia, 2021). A solid understanding of linear equations augments performance in other mathematical topics. However, despite their importance, Ghanaian Junior High School (JHS) students struggle with equations. The West Africa Examination Council reports from both 2011 and 2017 indicated that the majority of students were unable to solve linear equations in one variable. Fletcher (2018) also echoed these concerns about the students' weak performance. Hence, there is an exigent need to resolve students' difficulties with linear equations in one variable.

Several studies reveal the influence of teaching methods on students' accomplishments (Ganyaupfu, 2013; Isa, 2020). Currently, instruction based on multiple representations is regarded as crucial to classroom mathematics education (Moseley, 2005). Such teaching approaches present lessons in diverse forms (Goldin & Shteingold, 2001). As per Tripathi (2008), this approach aids students' comprehension and augments their mathematical performance. Other studies endorse the role of multiple representations in abstracting mathematical concepts and bolstering student learning and problem-solving (Ross & Willson, 2012). Thus, multiple representations-based instruction paves the way for students to construct and enhance their understanding of classroom instruction (Nabie, 2009).

Notwithstanding, despite the proven effectiveness of multiple representations-based teaching in improving student understanding, it remains underutilized in many mathematics classrooms, including in subjects like algebra and linear equations that employ a wide spectrum of multiple representations (Lubinski & Otto, 2002). Research findings indicate teachers' deficiencies in integrating multiple representations in teaching (Celik & Baki, 2007). Consequently, most concepts are taught via single representation, bypassing the benefits of multiple representations. Hitt (2001) highlighted that most educators prioritize a single mode of representation, often neglecting others. To address the issue of JHS students' poor performance in linear equations in one variable and mathematics in general in Ghana, an investigation into the teaching methods utilized by JHS mathematics teachers is imperative, as is the search for methods that can enhance student performance. This study aims to explore the methods employed by JHS mathematics teachers in teaching linear equations and evaluate the impact of multiple representation-based teaching on student achievement in linear equations in one variable within the Bimbilla Municipality of Ghana. The research questions guiding this study include:

1. Which modes of representation do teachers employ in teaching one-variable linear equations in the Bimbilla Municipality?
2. What factors influence teachers' choice of representations when teaching one-variable linear equations in the Bimbilla Municipality?

METHOD

Research Methodology

The researchers implemented a quasi-experimental research design to address the research questions and hypothesis, incorporating both quantitative and qualitative data analyses. A quasi-experimental design was deemed suitable as the researchers desired to study participants in their naturally formed classes or groups, without random assignment to experimental or control groups (Gall et al., 2007). This approach allowed for a detailed assessment of the approach's effects on participants in their typical class settings. The chosen design, frequently recommended for educational evaluations, serves as a viable alternative to randomized experiments (Schneider et al., 2007).

Among the many forms of quasi-experimental design, a non-equivalent control group design was employed. In this method, participants from both the experimental and control groups undertake pre-tests and post-tests, with only the experimental group receiving the treatment. This was chosen due to its popularity as a strategy within quasi-experimental design (Creswell, 1994).

Sampling Strategy and Participants

The study purposively selected three Junior High Schools from Bimbilla Municipality, on the grounds of their comparable levels of mathematical achievement. An opportune sample of 53 students was enlisted from each school (School A, B, and C), yielding a total of 159 participants across all three schools. A convenience sampling technique was adopted, allowing for the selection of readily available students who were willing to participate in the study. School A consisted of 24 boys and 29 girls, School B was made up of 16 boys and 37 girls, while School C included 28 boys and 25 girls. Additionally, 86 mathematics teachers were selected for the study as they could provide valuable insights relevant to the study's objectives, given their role in teaching the students linear equations in one variable (Tashakkori & Teddie, 2003). School representation was determined as follows: School A constituted experimental group 1, School B represented experimental group 2, and School C served as the control group.

Research Instruments

Given the study's mixed-method data analysis approach, questionnaires and a Linear Equations Achievement Test (LEAT) were used as research instruments for data collection. The questionnaire consisted of two sections: Section A gathered demographic details about the teachers, encompassing characteristics such as gender, age, professional status, qualifications, and tenure as a mathematics teacher. Section B explored the teachers' modes of representation in teaching linear equations. The LEAT, comprising a pre-test and post-test designed by the researchers, gauged students' achievement levels in one-variable linear equations.

Instrument Validity and Reliability

The validity of an instrument relates to the degree to which it accurately measures what it is intended to measure (Durrheim, 1999). In other words, an instrument is considered valid if it effectively fulfills its designated purpose. In contrast, reliability pertains to the consistency, trustworthiness, and reproducibility of the results procured from a research study (Numan, 1999). A measuring instrument exhibits reliability if it consistently produces the same results on multiple applications, or if the test scores show a high degree of consistency (Thompson & Durheim, 1999).

To ensure the validity of the equations achievement tests, the researchers referred to the Junior High School mathematics curriculum and textbooks endorsed by the Ghana Education Service during the test item development process. Questions from previous WAEC exams were also incorporated. To establish content validity, mathematics education specialists scrutinized and adjusted the questionnaire where necessary.

Following the pilot study of the instruments, the internal consistency of the questionnaire and the linear equations' achievement tests were determined by calculating reliability coefficients using the Cronbach alpha and Split-half method. The researchers used SPSS software to calculate the reliability coefficient for all instruments. With the Split-half method, student scores from the pre-test and post-test were divided into two equal halves and scored. The scores of odd-numbered and even-numbered items for each participant were then identified. The reliability coefficients were subsequently calculated using SPSS. The Spearman-Brown coefficient values for the pretest and post-test were 0.71 and 0.72 respectively. The questionnaire's reliability coefficient, denoted by Cronbach's alpha (α), was 0.76. Given these ranges, the reliability estimates were deemed satisfactory (Gearge & Mallery, 2003). As such, the instruments were regarded as reliable for the purposes of this study.

Data Collection Procedure

After obtaining the necessary permissions and approvals to conduct the study in the selected sample schools, the researchers administered a Linear Equations Achievement Test (pre-test) to the three distinct groups. The purpose was to gauge the students' existing understanding of linear equations in one variable. The completed tests were subsequently collected for processing and

analysis. Concurrently, teachers were given questionnaires designed to ascertain the methods they employed when teaching linear equations, including their reasons for choosing particular representations. The gathered data was then processed and analysed. Upon completion of the pre-tests for students and the questionnaires for teachers, the researchers initiated teaching activities for Experimental Groups 1 and 2, and the Control Group. Despite teaching the same topic with identical lesson objectives, the three groups received different teaching approaches. Experimental Group 1 was taught using manipulatives, graphic and algebraic representations. Experimental Group 2 was taught using manipulatives and algebraic representations, while the Control Group received instruction via algebraic representation alone.

The teaching plans formed the instructional design in this study, incorporating diverse activities to teach linear equations. These instructional designs were implemented in Experimental Groups 1 and 2, which had lessons scheduled on Mondays and Tuesdays, respectively, from 2:05pm to 2:45pm each week. The Control Group had their lessons on Wednesdays. The entire instruction period lasted for three weeks.

A researcher led a series of activities with students in Experimental Group 1, utilizing manipulative, graphic, and algebraic representations to teach linear equations in one variable. The concept of additive identity (or zero pair) was illustrated using tiles, indicating that any number or term and its opposite equate to zero. For example, -1 +1 = 0; x + (-x) = 0, and so on. The students were guided to understand that a red bar and a blue bar, or a red square and a blue square, both amount to zero. The rules of subtraction were also discussed, as in the case of 4 + (-2), where four blue squares and two red squares were represented. The principle of equality that any action performed on one side must be mirrored on the other to maintain balance was explained using tiles. For instance, the equation -3x + 2 = -4 was modelled using tiles, with two red squares added to both sides. Lastly, the students were shown that positive variables (x) were represented by blue, green, and yellow bars, positive constants by blue, green, and yellow squares, while negative variables (-x) were represented by red bars, and negative constants by red squares.

The researcher proceeded with the modelling of linear equations such as 3x+2=8+x, -2x-4x=2x-8, 4x+6=10, 3(x-2) =-4x+1, 3x-6=x+8, $\frac{1}{2}x$+5=2x+3, using manipulatives. The students were then guided to solve similar equations, leveraging the manipulative representation. The students modeled the equation 3x+2=8+x using tactile manipulatives. They were prompted to independently solve equations such as 4-x=2-3x, 4x+3=x+9, 2(x+3) =-3x-4 using these manipulatives. The researcher also facilitated group discussions among students to encourage peer-to-peer learning.

The students were instructed to graph simple linear equations in one variable, while the researcher explained the principle that the intersection point of the two lines on the graph corresponds to the truth set of the equations. For example, the equation 3x+2=8+x was graphed in the form y=3x+2 and y=8+x. The intersection point of these lines, which was 3, satisfied the truth set of the equation. The students were guided to repeat this process with the equation 4-x=2-3x, graphing it in the form y=4-x and y=2-3x. The intersection point -1 satisfied the truth set of the equation. The researcher concluded by instructing the students to algebraically represent the equations, without the use of manipulatives or graphs. The instruction then shifted to solving simple linear equations like 3x+2=8+x using an algebraic approach. The students were reminded of the equality principle, which dictates that any operation carried out on one side of an equation must be mirrored on the other to maintain balance.

Experimental Group 2 received similar instructions on linear equations in one variable, this time only with manipulative and algebraic representations, delivered by a second researcher. Additive identity (zero pair) was reintroduced using tiles, alongside a revision of the rules of subtraction. The equality principle was again emphasised, and the different colour representations for positive and negative variables and constants were reiterated.

The second researcher carried out a series of activities with the students in Experimental Group 2, guiding them to solve linear equations using manipulatives such as 3x+2=8+x, -2x-4x=2x-8, 4x+6=10, 3(x-2) =-4x+1, 3x-6=x+8, $\frac{1}{2}x$+5=2x+3. Further student-led discussions were encouraged to deepen their understanding. The instruction culminated in the students using algebraic representation to solve similar linear equations. The Control Group was taught linear equations in one variable exclusively through algebraic representation, under the guidance of a third researcher.

The teaching method was lecture-based, and examples were provided to guide students in using algebraic representation to solve a series of linear equations.

Following the introduction of the multiple representations-based instruction, a Linear Equations Achievement Test (Post-test) was administered to all three groups. This allowed for a measurement of students' progress in understanding linear equations in one variable after the treatment. The students were prompted to use various representations to solve the questions, thus testing the efficacy of the different teaching methods.

RESULTS AND DISCUSSION

Demographic Overview of Participating Teachers

This study, a comprehensive range of demographic data was compiled for the involved teachers, encompassing various characteristics such as gender identification, age, professional status, educational qualifications, and tenure as mathematics educators. The intent was to gain an in-depth understanding of the teachers participating in this research. The collated data on the demographic characteristics of the teachers are presented in Table 1.

Table 1. Summary of Demographic Characteristics of Teachers

Demographic factors	Category	Frequency	Percentage (%)
Gender	Male	68	79.1
	Female	18	20.9
Total		86	100.0
Age	20 – 25 years	7	8.1
	26 – 30 years	36	41.9
	31 – 35 years	28	32.6
	36 – 40 years	11	12.8
	41 years and above	4	4.7
Total		86	100.0
Professional Status	Pupil-teacher	1	1.2
	Non-professional	8	9.3
	Professional	77	89.5
Total		86	100.0
Academic Qualification	SSCE/WASSCE	1	1.2
	Certificate „A"	2	2.3
	Diploma	33	38.4
	HND	6	7.0
	Degree (B.SC. /Bed etc	38	44.2
	Masters	6	7.0
Total		86	100.0
Years of Teaching Mathematics	1 – 5 years	45	52.3
	6 – 10 years	26	30.2
	11 – 15 years	10	11.6
	16-20years	3	3.5
	21 years and above	2	2.3
Total		86	100.0

The data depicted in Table 1 concerning teachers' gender status reveals that out of the mathematics teachers participating in the study, 68 (79.1%) were male, while 18 (20.9%) were female. It's clear from this data that, within the Bimbilla Municipality at the time of this study, male teachers outnumber female teachers in Junior High School level mathematics teaching by a ratio of 75 more males. Focusing on the teachers' ages, the data demonstrates that the majority fell within the younger age ranges of 26-30 and 31-35 years, with these groups making up 36 (41.9%) and 28 (32.6%) of the teachers, respectively. Fewer teachers fell within the 20-25 age bracket and those aged 41 and above, with these categories accounting for 7 (8.1%) and 4 (4.7%) of the total teacher population, respectively. Regarding professional status, the majority of teachers, 77 (89.5%), held professional teaching credentials, with a single teacher identified as a pupil-teacher, making up 1.2% of the population. This signifies that the majority of teachers sampled were trained educators. The

academic qualifications of the teachers showed a fair spread. The data indicated that 33 (38.4%) of the teachers held Diplomas, while 38 (44.2%) had Bachelor's degrees. Only one teacher (1.2%) held an SSCE/WASSCE, with 6 (7.0%) holding Master's degrees, and another 6 (7.0%) with HND certificates.

Finally, when it comes to teaching experience, the data reveals that 45 (52.3%) teachers have been teaching mathematics for between 1 and 5 years, while 26 (30.2%) have been teaching for 6 to 10 years. This suggests that a significant portion of the teachers have relatively short to mid-length careers in mathematics teaching. Only 2 (2.3%) teachers have been teaching mathematics for more than 21 years.

Student Demographics

The demographic data for the students are outlined in Table 2.

Table 2. Summary of Demographic Characteristics of Students

Demographic factors	Category	Frequency	Percentage (%)
Gender	Male	68	42.8
	Female	91	57.2
Total		159	100.0
Age	13 – 15 years	109	68.6
	16 – 18 years	49	30.8
	19 years and above	1	0.6
Total		159	100.0

The data in Table 2 concerning students' gender indicates that out of the total students participating in the study, 68 (42.8%) were male, while 91 (57.2%) were female. It is clear from these figures that more female students participated in the study compared to male students.
Furthermore, in terms of students' age, the majority of students were between 13 and 15 years old, accounting for 109 (68.6%) of the participants. Only a single student, representing 0.6% of the sample, was 19 years or older. This suggests that the vast majority of the students sampled were within the 13 to 15 years age bracket.

Research question 1 results: *What forms of representation do teachers use in teaching linear equations in one variable within the Bimbilla Municipality?*

The intention behind Research Question 1 was to identify the types of representation employed by teachers in teaching linear equations in one variable. The data collected on the modes of representation used by teachers is outlined in Table 3. The responses were categorized as follows: Never (N), Almost Never (AN), Occasionally (O), Almost Every Time (AE), and Every Time (ET).

Table 3. Teachers' Mode of Representations on Linear Equations in One Variable

Description of mode of representations	N N (%)	AN N (%)	O N (%)	AE N (%)	ET N (%)	Mean	SD
Algebraic	0(0%)	0(0%)	0(0%)	29(33.7%)	57(66.3)	4.66	0.48
Manipulatives	32(37.2%)	29(33.7%)	4(4.7%)	15(17.4%)	6(7.0%)	2.23	1.31
Graphic	43(50.0%)	19(22.1%)	14(16.3%)	8(9.3%)	2(2.3%)	1.92	1.12
Multiple Representations	19(22.1%)	13(15.1%)	26(30.2%)	16(18.6%)	12(14.0)	2.87	1.34
Single Representation	0(0%)	2(2.3%)	11(12.8%)	34(39.5%)	39(45.3)	4.28	0.78

The delineations of modes of representation as presented in Table 3 were categorized by the researchers as the usage of transposition and similar techniques, the application of tiles and other auxiliary materials, teaching linear equations through graphical plotting, utilizing a blend of modes, and teaching linear equations with a singular representation.

The findings outlined in Table 3 suggest that a significant majority of the sampled teachers, 57 (66.3%), consistently used algebraic representation. None of the teachers reported never or almost never using algebraic representation in teaching linear equations. A smaller number of teachers affirmed the constant usage of manipulatives, graphical tools, and multiple representations, represented by 6 (7.0%), 2 (2.3%), and 12 (14.0%) teachers, respectively.

The results revealed that the majority of Junior High School mathematics teachers, 45.3%, regularly employed a singular representational approach in teaching linear equations. Conversely, a smaller group of teachers, 14.0%, consistently used multiple representational strategies. The predominant usage of the algebraic representation, employed by 66.3% of teachers, aligns with other research findings. For instance, Bal (2014) reported that algebraic representation is most commonly used in teaching linear equations.

Possible justifications for the preference for algebraic representation could be the perceived ease of understanding and the belief that it offers a faster method, as derived from data in Table 4.4. Conversely, the usage of manipulatives, graphical tools, and multiple representations were less popular among teachers. This supports the findings of Delice and Sevimli (2010), which suggested that the adoption of multiple representations was not as widespread as expected. Similarly, Gagatsis and Shiakalli (2004) found that graphical representation was the least used by teachers. This could be due to challenges teachers face in incorporating multiple representations into their instructional environment (Celik & Baki, 2007).

Research Question 2 Results: What influences teachers' choice of representation in teaching linear equations in one variable within the Bimbilla Municipality?

This research question was utilized by the researchers to identify the reasons why teachers elected to use or not use certain representations in teaching linear equations in one variable. The data derived from the questionnaire (open-ended section) are summarized in Table 4.

Table 4. Reasons for Teachers' mode of Representations in Linear Equations

Representations	Reasons	Frequency	Percentage (%)
Algebraic	Easy, simple and understandable	54	62.8
	Pupils have knowledge on it	8	9.3
	Well-known representation	23	14.0
	Faster	4	4.6
	Others	8	9.3
	Total	86	100.0
Manipulatives	Better understanding	13	52.0
	Makes lesson real	10	40.0
	Others	2	8.0
	Total	25	100.0
Graphic	Better understanding	10	41.6
	Makes lesson practical	7	29.9
	Others	7	29.2
	Total	24	100.0
Multiple represenation	Better understanding	17	31.5
	Motivates students	14	25.9
	Address different learning styles	12	25.2
	Others	11	20.4
	Total	54	100.0

The data in Table 4 reveals that among teachers who utilize algebraic representation, 54 (62.8%) reported that algebraic representation is straightforward, simple, and understandable, while 12 (14.0%) indicated that it is a well-known and commonly used representation method. When it comes to teachers who incorporate both manipulatives and algebraic representations in teaching linear equations, a majority stated that the use of two representations enhances understanding, representing 13 (52.0%) and 10 (41.6%) teachers respectively, for teachers who adopt multiple representations, 17 (31.5%) expressed that it fosters understanding and 14 (25.9%) stated that it engages and motivates students. However, teachers who opt for single representations offered similar reasoning to those who favor the algebraic approach. Furthermore, data regarding why teachers abstain from using certain representations in teaching linear equations were also gathered and are reported in Table 5.

Table 5. Reasons for teachers not using certain mode of representations in linear equations

Representations	Reasons	Frequency	Percentage (%)
Manipulatives	Time consuming	9	14.8
	Difficult to understand	10	16.8
	Lack of materials	19	31.1
	Have no idea	16	26.2
	Others	7	11.5
	Total	61	100.0
Graphic	Time consuming	13	21.0
	Difficult to understand	24	38.7
	Not found in syllabus	13	21.0
	Have no idea	8	12.9
	Others	4	6.5
	Total	62	100.0
Multiple represenation	Time consuming	13	40.6
	Lack of materials	8	25.0
	Students get confuse	3	9.4
	Have no idea	6	18.8.
	Others	2	6.3
	Total	32	100.0

The study findings presented in Table 5 reveal that, of the 61 teachers who refrain from using manipulatives to teach linear equations, 19 (31.1%) attribute their decision to a lack of available materials, and 16 (26.2%) indicate a lack of knowledge on how to incorporate manipulatives into their teaching. Other contributing factors, including cost, lack of recommendation in the syllabus, and irrelevance to pupils, among others, account for 11.5%. Regarding the 62 teachers who do not adopt graphic representation, 24 (38.7%) express that it is difficult for their students to comprehend, while 13 (21.0%) cite reasons such as time constraints and lack of inclusion in the syllabus. Additional reasons, such as cost, lack of materials, and examination questions not set on it, among others, account for 6.5%. Out of the teachers who eschew multiple representations, 13 (40.6%) argue that it's too time-consuming, while 8 (25.0%) cite lack of materials. However, for the algebraic representation, nearly all teachers report utilizing it, therefore providing no reasons for abstention. The same applies to single representation.

The findings underscore that the majority of teachers favor algebraic representation when teaching linear equations, with reasons such as simplicity, speed, widespread use, and comprehensibility. Consequently, these factors influence the teachers' choice of algebraic representation. This outcome aligns with Bal (2014) finding where teachers justified their preference for algebraic representation as it was understandable. The likely explanation is that most teachers believe their students grasp linear equations more readily when instructed via algebraic representation. Similarly, the study indicates that teachers steer clear of manipulatives, graphic, and multiple representations due to constraints like time, lack of materials, limited ideas, and difficulty for students. Bal (2014) also concluded that teachers' use of representations hinges on their understanding - if a teacher lacks insight into a particular representation, they are unlikely to use it. It was observed that only a small number of teachers employed manipulatives, graphic, and multiple representations when teaching linear equations.

Results of Research Hypothesis

H_0: No statistically significant difference exists between the scores of students exposed to multiple representation-based instruction and single representation-based instruction in linear equations achievement tests.

H_1: A statistically significant difference exists between the scores of students exposed to multiple representation-based instruction and single representation-based instruction in linear equations achievement tests.

Prior to testing the research hypothesis, the descriptive statistics of students' pre-test and post-test scores were calculated and are presented in Table 6.

Table 6. Mean Scores of Students' Pre-test and Post-test by Group

Variable	Group	Mean	SD	Min	Max
Pre-test	Exp group 1	3.13	1.13	1	6
	Exp group 2	3.02	1.84	0	9
	Cont. group	2.91	1.04	1	6
Post-test	Exp group 1	4.11	1.71	1	9
	Exp group 2	3.23	2.30	0	9
	Cont. group	2.51	1.42	0	6

The descriptive statistics presented in Table 6 demonstrate that the mean pre-test scores for students in Experimental Group 1 (M = 3.13, SD = 1.13) rose in the post-test (M = 4.11, SD = 1.71). Similarly, the pre-test mean scores for Experimental Group 2 (M = 3.02, SD = 1.84) also increased in the post-test (M = 3.23, SD = 2.30). In contrast, the Control Group's mean pre-test scores (M = 2.91, SD = 1.04) did not rise in the post-test (M = 2.51, SD = 1.42). Both Experimental Group 1 and the Control Group achieved the same minimum and maximum values (1 and 6) for the pre-test. Meanwhile, in the post-test, the maximum scores for Experimental Group 1 and Experimental Group 2 reached 9. This data signifies that the lowest and highest pre-test scores achieved by students in both Experimental Group 1 and the Control Group were identical. However, after the teaching period, students in the experimental groups recorded the highest post-test scores, indicating a notable improvement. To test the research hypothesis, the study utilized one-way Analysis of Covariance (ANCOVA). The assumptions considered included Independency of observation, Normality, Measurement of the covariates, Reliability of the covariates, correlation between the covariates and the dependent variable, Linearity, Homogeneity of variance, and Homogeneity of regression (slopes).

The research team personally administered and supervised the pre-tests and post-tests, ensuring that each student completed the questions independently. Consequently, Independency was observed. The assumptions of Normality were examined for students' pre-test and post-test scores using a histogram with a normality curve. The covariates were measured prior to the intervention. The pre-test registered a reliability coefficient of 0.71. The results of the correlation between the covariates and the dependent variable (students' post-test scores) were evaluated and are presented in Table 7.

Table 7. Correlations Between The Covariates and The Dependent Variable

Students Pre-test Scores		Students' Post-test scores
	Pearson Correlation	0.512**
	Sig. (2-tailed)	0.000
	N	159
Students' Gender	Pearson Correlation	-.131
	Sig. (2-tailed)	0.100
	N	159
Students' age in years	Pearson Correlation	0.170*
	Sig. (2-tailed)	0.032
	N	159

The correlation results presented in Table 7 indicate a positive correlation between students' pre-test and post-test scores, with r (159) = 0.51, P < 0.01. According to Cohen (1988), this correlation range (r = 0.51) is considered a medium or reasonable correlation. Meanwhile, the correlation between students' age and post-test scores is small, r (159) = 0.17, P < 0.05. However, the correlation between students' gender and post-test scores did not reach a significant level, r (159) = -0.13, P = 0.10. Therefore, students' age and gender were not included as covariates. In line with the assumption testing, the researchers confirmed the linearity between students' pre-test and post-test scores, students' age and post-test scores, and students' gender and post-test scores across the groups, corroborating the evidence presented in Table 8. Furthermore, the researchers were required to test the assumption of homogeneity of variance to verify that the variance of post-test scores across the groups is equal. The results of this test are presented in Table 8.

Table 8. Levene's Test of Equality of Error Variance

F	df1	df2	Sig.
2.784	2	156	0.065

The results in Table 8 demonstrate that the assumption of homogeneity of variance has not been violated, as indicated by F (2, 156) = 2.784, with P > 0.05. This indicates that the variance of post-test scores across the groups is equal, thus confirming that the assumption is met. Subsequently, the researchers examined the assumption of homogeneity of regression. This was done to ensure that there isn't an interaction between the pre-test scores and the groups. The results of this examination are presented in Table 9.

Table 9. Interaction Between The Pre-Test Scores and The Groups

Source	Type Sum of Squares	Df	Mean Square	F	Sig
Group	5.909	2	2.955	1.011	0.366
Pretest	61.731	1	61.731	21.120	0.000
Group*Pretest	0.267	2	0.134	0.046	0.955
Error	447.202	153	2.923		
Corrected Total	515.109	158			

The results in Table 9 demonstrate that the assumption of homogeneity of regression has not been violated, as indicated by F (2,153) = 0.046, with p > 0.05. This indicates that there isn't an interaction between pre-test scores and the groups, thus confirming that the assumption is met. The researchers then conducted the One-way Analysis of Covariance (ANCOVA) after all necessary assumptions were satisfied in order to test the research hypothesis. The independent variable, referred to as 'group', consisted of three levels: Experimental group 1, Experimental group 2, and the Control group. The students' pre-test and post-test scores were used as the covariate and dependent variable respectively. The ANCOVA results are reported in Table 10.

Table 10. Results of ANCOVA for Post-test Scores of Students

Source	Type Ш Sum of Squares	Df	Mean Square	F	Sig	Partial Eta Square
Pretest	82.339	1	82.339	28.521	0.000	0.155
Group	58.897	2	29.449	10.201	0.000	0.116
Error	447.470	155	2.887			
Corrected Total	588.706	158				

The statistics in Table 10 demonstrate that after verifying the assumption of homogeneity of regression to ensure no interaction between pre-test scores and groups F (2,153) = 0.046, P > 0.05, and observing a significant relationship between pre-test scores and posttest scores F(1,155) =28.52, p < 0.05, η^2=0.16), the ANCOVA results revealed a statistically significant group difference in students' achievement scores (F(2,155) =10.20, p < 0.05, η^2=0.12). This means that 12% of the variance in post-test scores can be explained by the intervention, which is considered a large effect size according to Cohen's (1988) criterion. To determine the specific group differences, a follow-up test based on LSD pairwise comparisons among the adjusted means was carried out. The results obtained are reported in Table 11.

Table 11. ANCOVA Pair Wise Comparisons of The Adjusted Means Among The Groups

Groups	Adjusted Mean	Comparisons	Mean Difference	Std. Error	Sig.
Exp. group 1	4.11	Exp. group 1 Vs. Exp. group 2	0.88*	0.330	0.014
Exp. group 2	3.23	Exp. Group 1 Vs. Cont. group	1.60*	0.329	0.000
Control group	2.51	Exp. Group 2 Vs. Cont. group	0.72*	0.332	0.048

* The mean difference is significant at the 0.05 level.

The statistics in Table 11 indicate that students in Experimental group 1 had the highest adjusted mean (M = 4.11), followed by Experimental group 2 (M = 3.23), with the Control group recording the lowest mean score (M = 2.51). Furthermore, the LSD pairwise comparisons of the adjusted means among the groups demonstrated that the mean difference in students' achievement scores was statistically significant between Experimental group 1 and Experimental group 2 (M = 0.88, P = 0.014), Experimental group 1 and Control group (M = 1.60, P = 0.000), and Experimental group 2 and Control group (M = 0.72, P = 0.048), after controlling for the effects of students' pre-test scores.

Based on the results of the ANCOVA analysis F (2,155) = 10.20, P = 0.05, η^2= 0.12, the null hypothesis was rejected in favor of the alternative hypothesis. This indicates that a statistically significant difference exists between students' scores on the linear equations achievement test when using multiple representations-based instruction compared to single representation instruction, after controlling for the effects of students' pre-test scores. The improvement in students' achievement in linear equations was attributed to the introduction of multiple representations-based instruction.

Upon analyzing students' scores in the linear equations achievement test based on the ANCOVA results, a statistically significant difference was found among the groups, favoring the multiple representations-based instruction F (2,155) = 10.20, P = 0.05, η^2= 0.12. The mean difference of the adjusted means among the groups was also significant. Students in Experimental group 1, who had the opportunity to learn linear equations through three different representations (manipulatives, graphic, and algebraic representations), achieved the highest performance as indicated by the highest adjusted mean. Students in Experimental group 2, who learned linear equations through two different representations (manipulatives and algebraic representation), also performed well, as indicated by the second-highest adjusted mean.

However, students in the Control group recorded the lowest performance, as indicated by the lowest adjusted mean. This finding aligns with a study conducted by Cikla (2004), suggesting that the utilization of multiple representations in teaching mathematical concepts can enhance students' achievement.

CONCLUSIONS

The research results indicate that students exhibit a statistically significant difference between their scores on the linear achievement test when utilizing dual-representation-based instruction compared to single-representation instruction. It is also primarily shown that a majority of middle school mathematics instructors (45.3%) consistently apply a single representational learning approach in their linear equation pedagogy. However, a small fraction of teachers (14.0%) regularly incorporate multiple representations in their teaching. This demonstrates that dual-representation-based learning influences middle school students' academic performance in single-variable linear equations.

Research on dual-representation-based learning in single-variable linear equations can provide valuable contributions towards a deeper understanding of more effective and relevant learning methods in the context of middle school mathematics education. Obstacles such as time management, student comprehension challenges, scarcity of resources, a lack of innovative strategies, and questions about relevance are some of the factors hindering the broader adoption of multi-representational methods. Nevertheless, it should be noted that the implementation of dual-representation-based instruction supports students' academic performance in the subject of single-variable linear equations.

AUTHOR CONTRIBUTION STATEMENT

PMT : Idea, desain, conceptualizon, analysis, and editing
RKM : Drafting the manuscript, correction, directing, and final approval
RAG : Editing, reviewing, and proofreading.

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